Fragmented duopoly
Theory and applications to backward agriculture*

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Abstract: The central aim of this paper is to provide a formal description of the idea of 'market fragmentation', which is so widely used to describe markets in backward agriculture. We begin by describing a 'fragmented duopoly' in which each seller has a captive set of customers and there is a non-captive set who may buy from either seller. The Nash-Cournot equilibria of the system are analyzed; and then the model is generalized to allow for the endogenous determination of the captive sets. The subgame perfect equilibrium of the model provides a useful background for studying familiar topics like fragmentation, interlinkage and disguised unemployment from a new point of view.

1. Introduction

In certain trades trust is a precondition for exchange or transaction to occur. This would be true where information asymmetries are strong. In buying used cars most of us would prefer to make a purchase from friends and acquaintances (or at least from some of them!). It is well known that in informal credit markets, where formal legal institutions are weak, a person would lend money only to those whom he can trust or over whom he has some control. Thus a landlord may agree to lend money only to his laborers and a merchant may agree to lend only to his regular customers. This has led to a view that credit markets are 'fragmented'. ¹ However, when it has come to actually modelling such a case the usual recourse has been to treat it as a case of several monopoly islands. Strictly speaking, however, the market just described is neither a monopoly nor a duopoly since the set of

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potential borrowers of the landlord would, typically, have some intersection with the set of potential borrowers of the merchant but the two sets would not be identical. What we have is a case in between a monopoly and oligopoly. It is this 'in between' case that is formally characterized and explored in this paper.

Let us assume that there are \( n \) sellers of a certain commodity. Let \( S_i \) be the set of potential customers of seller \( i \). To consumers outside \( S_i \), \( i \) will never sell goods, irrespective of the price. Consider now two special cases. First, if it is true that

\[ S_1 = S_2 = \cdots = S_n, \]

then we have a case of standard oligopoly with \( n \) firms. All firms are competing over the same set of customers.

If, on the other hand, \( (S_1, S_2, \ldots, S_n) \) happens to be a partition over the set of all potential customers in the economy, then we have a case of \( n \) standard monopolies. Each seller has his own exclusive pool of customers.

There is no reason why we have to restrict attention to these two polar cases. We may well have cases where for some \( i, j \), the sets \( S_i \) and \( S_j \) have some common members but it is not the case that \( S_i \) is the same set for all \( i \). We shall describe a market structure where this happens (along with the two polar cases just described) as a fragmented oligopoly.

Though we motivated the idea of fragmented oligopoly by talking about the role of trust and control in certain transactions,\(^3\) we believe that this market structure could be usefully applied in many different areas. It clearly has relevance to models of industrial location. Indeed, certain features of location contribute to the fragmentation of rural credit markets when the pattern of settlement is nucleated, as in the case of South Asia's villages, rather than continuous, as in Hotelling's (1929) classic work. It is known, for example, that not all villages have resident moneylenders [Reserve Bank of India (1954)] and that commission agents and traders often have 'territories' made up of several contiguous villages from which most of their clients in moneylending and trade are drawn.\(^4\) Drawing upon these examples, suppose that there are three villages, A, B and C, in a row. Moneylender 1 lives in village A; and moneylender 2 lives in C. If we suppose that the inhabitants of A would go only to their 'resident' moneylender and likewise for C, and that those of B would go to whoever charges less, then we have a

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\(^2\)Such a model is developed in Basu (1987) where the rural credit market is modelled as a collection of independent credit islands.

\(^3\)Trust plays an important role not only in backward markets but in a whole range of interactions in any economy: see Dasgupta (1986).

\(^4\)This was revealed in conversations between commission agents and Bell in the course of fieldwork in Andhra Pradesh and Punjab, India.
case of fragmented duopoly. If \( N_X \) is a set of people in village \( X \), then this is a special case of the above formal definition with \( n=2 \) and \( S_1 = N_A \cup N_B \) and \( S_2 = N_C \cup N_D \). Thus, though the model that we construct does not belong to the class of location models based on Hotelling's [see, for example, D'Aspremont, Gabszewicz and Thisse (1979) and Bonanno (1987)] and the properties that we investigate are distinct from the ones that a model of location would focus on, the abstract structure could be used as a basis for a model of locational duopoly.

Another view of our model is that of an oligopoly with switching costs [e.g., von Weizsäcker (1984); Klemperer (1987a, b); and Bulow, Geanakoplos and Klemperer (1985)]. Indeed our model may be viewed as an application of switching cost theory, with prohibitive switching costs once the 'domains' or 'territories' of firms have been established, to the study of backward markets and agrarian relations. Though our initial model, in abstraction, is a kind of switching-cost model, we develop it in some detail as our aim is to address issues in development and to persuade development economists of the relevance of such models of industrial organization to agrarian theory.

Models of fragmented oligopoly could also find application in activities where because of asymmetric information each seller has a predetermined clientele that trusts him. International trade with prior political fragmentation is another area of possible application of this theory. Though this paper is an abstract analysis of fragmented duopoly and little depends on what actual motivation is used, our interest in the subject arose from an attempt to give a rigorous characterization of the idea of 'market fragmentation' which is so central to development economics and particularly the theory of agrarian structure. It is for this reason that much of the paper dwells on problems of backward agriculture.

When firms possess captive segments of the market, it is natural to ask whether they can practice price discrimination between segments. This is indeed an open question. It is arguable that in fragmented agrarian markets, which are our central concern in this paper, arbitrage is not easy and so price discrimination ought to be treated as feasible. One must, however, remember that in personalized rural markets of the kind described in Bardhan (1984), the possibility of price discrimination may be thwarted by social norms. In different societies, different kinds of discrimination are treated by the people as 'unjust'. The origins of these norms lie in distant history but are often powerful enough to make certain kinds of discriminatory pricing infeasible. That is, the cost in terms of political dissension is too high from the seller's point of view. It is for this reason that we have in this paper devoted somewhat more attention to the non-discrimination model. We do, however, deal with the case of segment-specific price discrimination in separate sections.

The Cournot–Nash equilibria of a fragmented duopoly in which sellers
cannot practice price discrimination are analyzed in section 2. Section 3 briefly describes the case where a seller can price-discriminate between market segments. In section 4 a two-period model is constructed in which in the first period the players fight to establish their domains, that is, the $S_1$'s and the $S_2$'s. In the second period they treat $S_1$ and $S_2$ as given and play a quantity-setting game. The subgame perfect equilibria of such a two-period game, with and without price discrimination, are examined. The case with price discrimination is taken up in section 5, and the possibility of rent-dissipation in this setting in section 6.

2. The Nash equilibrium of a fragmented duopoly without price discrimination

There are $n$ identical consumers and each consumer’s demand function for the commodity in question is given by

$$q = q(p),$$

where $p$ is price and $q$ is quantity demanded. We assume $q$ is a continuous function and $q'(p) < 0$. The inverse demand function is written as follows:

$$p = p(q).$$

There are two sellers (or firms), 1 and 2. The $n$ consumers are partitioned into three sets, $N_1$, $N_2$ and $N_3$, consisting of, respectively, $n_1$, $n_2$ and $n_3$ persons. Thus $n_1 + n_2 + n_3 = n$. The members of $N_1$ would buy goods from only firm 1. Members of $N_2$ would buy from only 2. The third group would buy from whoever offers better terms. These three groups will be referred to as the three segments of the market. $N_i$ is firm $i$’s captive segment, for $i = 1, 2$; and $N_3$ will be referred to as the contested segment.\(^5\)

Both firms have the same cost function: A cost of $c$ units has to be incurred to produce each unit of the good. An immediate consequence of this assumption is that were the firm able to charge the monopoly price, $p^m$, in its captive segment of the market, $p^m$ would depend only on the shape of the individual’s demand function and $c$. That is, $p^m$ is then independent of the pattern of market segmentation. This is taken up further in sections 3 and 5.

Consider now firm $i$’s problem. It has to decide how much to supply to its captive segment, $x_i$, and how much to apply to the contested segment, $q_i$. To begin with, it will be assumed that a firm cannot discriminate between consumers in terms of the price charged. The consequence of relaxing this assumption is discussed in section 3.

Suppose each firm has chosen a strategy. That is, we are given $(x_1, q_1, x_2, q_2)$. Clearly the price of the good in the contested segment will be

\(^5\)This ought not to be confused with the concept of ‘contestable’ markets in the literature.
If \( q_i > 0 \), then the fact that a firm must charge the same price to all customers means that \( i \) must charge a price of \( p((q_1 + q_2)/n_3) \) even in its captive segment. As price depends on the other firm’s choice of \( q_j \), the firm’s choice of \( x_i \) may not be consistent with demand in the captive segment of its market. Thus \( i \)’s total profit will be

\[
p = p(q_1 + q_2)/n_3.
\]

Note that \( n_i q(p((q_1 + q_2)/n_3)) \) is the demand for the good in the captive segment when price is \( p((q_1 + q_2)/n_3) \); and the shorter side of the market determines the volume of sales when supply is not equal to demand.

It is easy to see, however, that, given \( q_1 \) and \( q_2 > 0 \), firm \( i \)’s choice of \( x_i \) can be deduced therefrom. Hence, we may define the profit of each firm in terms of only \( q_1 \) and \( q_2 \). Using \( \pi_i \) to denote firm \( i \)’s profit, we have:

\[
\pi_i(q_1, q_2) = \begin{cases} 
\max_{x_i} \left[ p \left( \frac{q_1 + q_2}{n_3} \right) - c \right] x_i & \text{if } q_i = 0 \\
\left[ p \left( \frac{q_1 + q_2}{n_3} \right) - c \right] \left[ q_i + \frac{n_i}{n_3} (q_1 + q_2) \right] & \text{if } q_i > 0.
\end{cases}
\]

The interpretation of this profit function is as follows. Given \((q_1, q_2)\), firm \( i \) supplies to its captive segment a profit-maximizing amount of goods. That is, given \( q_1 \) and \( q_2 \), price is determined by \( p((q_1 + q_2)/n_3) \); so that if \( q_i > 0 \), then in its captive segment, firm \( i \) supplies exactly the amount that is demanded, which is equal to \((n_i/n_3)(q_1 + q_2)\). If \( q_i = 0 \), then the fact that a firm has to charge the same price to all its buyers places no restriction on the price it can charge in the captive segment. In such a case it can charge the monopoly price, \( p^m \), and make monopoly profits. These features are captured in (4).

Let us define the **Nash equilibrium** of the game as a \((q_1^*, q_2^*)\) such that \( \pi_1(q_1^*, q_2^*) \geq \pi_1(q_1, q_2^*) \) for all \( q_1 \) and \( \pi_2(q_1^*, q_2^*) \geq \pi_2(q_1^*, q_2) \) for all \( q_2 \).

It is useful to have a visual representation of the reaction functions. This would enable us to compare a fragmented duopoly with a traditional duopoly. In fig. 1, firm 1’s reaction functions lie in the NE- and NW-quadrants. If \( q_2 \) is zero, firm 1 acts as a monopolist on both its captive

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\( ^6 \)We assume throughout that \( \max_{x_i} [p(x_i/n_i) - c] x_i > 0 \), for \( i = 1 \) or \( 2 \). This ensures that production is profitable.
segment and the contested segment. In that case, let $OC$ be the amount it supplies on its captive segment and $OA$ be the amount it supplies on the contested segment. As $q_2$ rises, firm 1's sales on the contested and captive segments are represented by the lines $AB$ and $CD$. It will be shown later that $AB$ will be steeper than the 45° line and $CD$ will be a rising curve, as shown. As $q_2$ rises, supply in firm 1's captive segments deviates more and more from the monopoly output $OC$. This happens because a firm has to charge the same price to all customers. As $q_2$ keeps rising, a point will be reached where firm 1 would prefer to drop out of the contested market and sell the monopoly output to its captive segment. This happens when $q_2 = OG$. For all $q_2 > OG$, firm 1 sells $GE = OC$ units on its captive segment and zero in the contested market.

We have here drawn a case which gives a 'stable' Nash equilibrium [in the sense of Friedman (1977)]. It is later shown that this must always be the case. In fig. 1 we also depict firm 2's reaction functions. These are denoted by the same letters with primes on them. The above discussion makes it clear that for analyzing the Nash equilibria of a fragmented duopoly we could concentrate exclusively on the NE-quadrant because the reactions in the captive segments (i.e., the NW- and SE-quadrants) could be derived mechanically from the happenings in the NE-quadrant.

In the case depicted in fig. 1 there is only one Nash equilibrium. But since the reaction functions have breaks it appears as if we can have corner equilibria as in models with fixed costs [e.g., Spence (1979), Dixit (1979, 1980), Basu and Singh (1990)], as illustrated in fig. 2. If $E_1$ had occurred, then firm 1 would be selling $OE_1$ on the contested segment, whereas firm 2...
would be selling only to its captive segment. It can, however, be shown that such equilibria can never arise in this model and in order to have such corner equilibria it may be necessary to introduce some fixed costs [Mishra (1991)]. It seems to us that in practice, markets do often get partitioned into zones within which each firm acts like a monopolist.\(^7\) However, as things stand, equilibrium is always unique and occurs with both firms supplying to the contested segment.

Let us now take note of a property of the reaction functions which has important implications for our model. Let the function \(R_i(q_j)\) denote firm \(i\)'s optimal [in terms of (4)] choice of \(q_i\) given that the other firm has chosen \(q_j\). In case there is more than one \(q_i\) which satisfies this condition, we shall assume that \(R_i\) specifies the smallest of these \(q_i\)'s. Thus in fig. 1, if \(q_2 = OG\), then \(R_1(q_2) = 0\).

Suppose \(q_2\) is such that \(R_1(q_2) > 0\). Hence in maximizing \(\pi_1\) we could [in eq. (4)] concentrate on the case where \(q_1 > 0\). That is,

\[
\pi_1(q_1, q_2) = \left[ p \left( \frac{q_1 + q_2}{n_3} \right) - c \right] \left[ q_1 + \frac{n_1}{n_3} (q_1 + q_2) \right].
\]

Maximizing \(\pi_1\) with respect to \(q_1\) gives us the following first-order condition:

\[
\frac{\partial \pi_1}{\partial q_1} = (p - c) \left( 1 + \frac{n_1}{n_3} \right) + \frac{p'}{n_3} \left[ \left( 1 + \frac{n_1}{n_3} \right) q_1 + \frac{n_1}{n_3} q_2 \right] = 0.
\] (5)

\(^7\)Recall that in this model both monopolists will charge the same price, since customers are identical. If, however, we allow for heterogeneity among customers, then the prices may be different.
This yields the following theorem.

**Theorem 1.**

If \( R_1(q_2) > 0 \), then \( \partial R_1/\partial q_2 > -1 \).

If \( R_2(q_1) > 0 \), then \( \partial R_2/\partial q_1 > -1 \).

**Proof.** See appendix.

Theorem 1 asserts that as long as both firms are operating on the contested segment, a decrease in \( q_2 \) by one unit causes an increase in \( R_1(q_2) \) by less than one unit; symmetrically, the same applies to firm 2's reaction function.

Using Theorem 1 we can quickly establish some corollaries. Observe that our Nash equilibrium must be stable in the sense that the reaction functions intersect in the 'correct' direction. This is because Theorem 1 implies that in the NE-quadrant of fig. 1, firm 1's reaction function must be steeper (in magnitude) than firm 2's reaction function.

**Corollary 1.** As long as firm 1 operates on the contested market, a fall in \( q_2 \) would cause firm 1's supply on its own captive segment to fall.
claim, along with Corollary 2 below, tells us that, in some sense, a fragmented duopoly lies where we expect it to lie – somewhere between a standard duopoly and a monopoly.

Let us now turn to the properties of the Nash equilibrium and do some comparative statics. If with \( n_2 \) and \( n \) remaining constant \( n_1 \) increases, what happens to firm 1’s share in the contested market? In other words, how does the size of one’s captive segment affect one’s share in the contested segment? The next theorem asserts that this relationship is a negative one. There is here an interesting analogy with Fudenberg and Tirole’s (1986, pp. 23–24) analysis in which an incumbent firm, planning to deter entry, prefers not to have a large captive segment.

**Theorem 2.** If \( n_1 \) increases with \( n \) and \( n_2 \) constant, then a Nash equilibrium, \((q_1, q_2)\), where \((q_1, q_2) > 0\), changes such that 1’s relative market share in the contested segment falls (i.e., \( q_1/q_2 \) falls).

Since we are looking at a case where in the Nash equilibrium \((q_1, q_2) > 0\), \((q_1, q_2)\) must satisfy (5) and, by symmetry, the following:

\[
(p - c) \left( 1 + \frac{n_2}{n_3} \right) + \frac{p'}{n} \left[ \left( 1 + \frac{n_2}{n_3} \right) q_2 + \frac{n_2}{n_3} q_1 \right] = 0. \tag{6}
\]

Eqs. (5) and (6) imply

\[
\frac{n_1 + n_3}{n_2 + n_3} = \frac{(n_1 + n_3)q_1 + n_1q_2}{(n_2 + n_3)q_2 + n_2q_1}.
\]

Cross-multiplying and substituting \( n - n_1 - n_2 \) for \( n_3 \), we get

\[
\frac{q_1}{q_2} = \frac{n - n_1}{n - n_2}. \tag{7}
\]

The theorem is immediate.

Just before stating the theorem we claimed that we were going to look, into the effect of \( n_1 \) on the market share. Clearly, this can be interpreted in several ways. What Theorem 2 examined was the effect of \( n_1 \) on \( q_1/q_2 \) with \( n_2 \) and \( n \) held constant. What, it may be asked, will be the effect of raising \( n_1 \) on \( q_1/q_2 \) if \( n_2 \) and \( n_3 \) are held constant? To answer this, note that (7) implies \( q_1/q_2 = (n_2 + n_3)/(n_1 + n_3) \). Hence, firm 1’s market share falls if \( n_1 \) increases and \( n \) is also increased by the same amount.

Eq. (7) tells us more than Theorem 2. Market shares in the contested
segment in a Nash equilibrium are independent of \( c \) and approach equality as \( n \) increases with \( n_1 \) and \( n_2 \) constant.

**Theorem 3.** The equilibrium price does not vary with changes in \( n_1 \) and \( n_2 \) as long as \( n_1 + n_2 \) and \( n \) remain unchanged, excepting in the special case where the change in \( n_1 \) and \( n_2 \) causes a firm to enter or withdraw from the contested market.

This is a somewhat surprising result. It asserts that in the determination of the industry's price and output, what matters is how much of the market is contested and how much captive. Excepting the special case mentioned in the theorem, the exact break-up of the total captive segment into firm 1's and 2's segments is inconsequential.

To prove this, consider first an 'interior solution', i.e., \((q_1, q_2) > 0\). Rewriting (5) and (6) and using (7), we get

\[
\frac{p'q_1}{p-c} = \frac{-(n-(n_1+n_2))(n-n_1)}{n}, \tag{8}
\]

\[
\frac{p'q_2}{p-c} = \frac{-(n-(n_1+n_2))(n-n_2)}{n}. \tag{9}
\]

Writing \( z \) for \((q_1 + q_2)/(n-(n_1 + n_2))\), from (8) and (9) we get

\[
z = \frac{p(z)-c}{p'(z)} \cdot \frac{2n-(n_1+n_2)}{n}. \tag{10}
\]

Eq. (10) implies that \( z \) will be unchanged as long as \( n, n_1 + n_2 \) and \( c \) remain unchanged. Since price depends on \( z \), price remains unchanged as long as \( n, n_1 + n_2 \) and \( c \) remain unchanged.

If we have a corner solution, the monopoly price will prevail no matter how the segmentation occurs.

It is interesting to observe that (10) implies that we cannot predict the direction of changes in price induced by changes in \( n \) or \( n_1 + n_2 \), unless we impose restrictions on \( p'' \). It is easily checked using (10) that a rise in \((n_1 + n_2)\) will cause \( z \) to fall, and hence price to rise, if \( p'' \leq 0 \). In other words, as the contested segment becomes smaller, \( p'' \leq 0 \) is a strongly sufficient condition for the equilibrium price to rise towards the monopoly price. As a standard duopoly is characterized by \( n_1 = n_2 = 0 \), we also have.

**Corollary 2.** If \( p'' \leq 0 \), industry output (price) in a fragmented duopoly will be less (greater) than that in a standard duopoly.
It is possible to use this model to do more comparative statics exercises and deduce other properties, but that is not the aim here. Instead we apply the theory of fragmented duopoly to a problem in backward agriculture and, in that context, explore how the captive segments of the market are established in a two-period setting. But before doing so, we make a brief digression to show how our model may be adapted to allow segment-specific price discrimination.

3. Fragmented duopoly with price discrimination

Let us assume, as before, that seller 1 chooses $q_1$ and $x_1$ but that he is free to set the price in his captive market, wherever he wishes. Given profit-maximizing behavior and given $q_1$, $q_2$, $x_i$, firm $i$ will set a price of $p(x_i/n_i)$ in its captive market, and in the contested market price will be given by $p((q_1 + q_2)/n_3)$. If we continue to assume a constant marginal cost, then all the segments get completely dismembered and equilibrium can be worked out separately for each. The monopoly price, $p^m$, will be charged in each of the captive segments, while the standard duopoly price will hold in the contested segment.

The problem is much more interesting if we suppose that firm $i$'s total cost function, $c_i(\cdot)$, is increasing and convex. Firm $i$'s profit, $\hat{\pi}_i$, is given as follows:

$$\hat{\pi}_i(q_1, q_2, x_i) = p\left(\frac{x_i}{n_i}\right) \cdot x_i + p\left(\frac{q_1 + q_2}{n_3}\right) q_i - c_i(q_i + x_i), \quad i = 1, 2.$$  

Firm $i$ maximizes this by choosing $q_i$ and $x_i$. Its first-door conditions are:

$$\frac{x_i}{n_i} p'\left(\frac{x_i}{n_i}\right) + \left(\frac{x_i}{n_i}\right) = c'_i(q_i + n_i) \quad (11)$$

and

$$\frac{q_i}{n_3} p'\left(\frac{q_1 + q_2}{n_3}\right) + \left(\frac{q_1 + q_2}{n_3}\right) = c'_i(q_i + n_i). \quad (12)$$

The Nash equilibrium of a price-discriminating fragmented duopoly is given by the $(x_1, q_1, x_2, q_2)$ derived from solving the four equations described by (11) and (12) and by setting $i$ equal to 1 and 2.

Comparative-statics results may be derived in much the same way as in the previous section. For example, assuming that the marginal revenue curve is downward sloping [i.e., the left-hand term in (11) falls as $x_i$ rises and the left-hand term in (12) falls as $q_i$ rises], Corollary 1 can be derived even for
the price-discrimination model. Moreover, following Bulow, Geanakoplos and Klemperer (1985), this model can be used to illustrate some surprising results, like how a subsidy in the captive segment of a seller can actually result in the seller being worse off in equilibrium. With these remarks we turn to the analysis of agrarian relations and the determination of the size of the captive and contested segments. We return to the subject of price discrimination in section 5.

4. Subgame perfection in a two-period model of agrarian relations

Since $n_1$ and $n_2$ influence the outcome of the one-period fragmented duopoly described in sections 2 and 3, it is but natural that firms will try to influence $n_1$ and $n_2$ to the extent that they can. For the sake of illustration consider a rural economy with $n$ laborers and two landlords. In period 1 each landlord $i$ decides on the number, $n_i$, of laborers he will employ on his land. In period 2 the landlords supply credit to them and to the contested segment of the market for loans. This periodization reflects more the priorities of decision than the actual sequence of time. Moreover, in reality period 2 will be further split up involving a first sub-period when the loan is received by the laborers and a second sub-period when the wage is received and the principal and interest is repaid. We, however, ignore this further temporal partitioning of period 2.

We shall assume - and this is not unrealistic - that each landlord has the power to forbid his employees from taking credit from the other landlord. Further, this is in a setting where everybody knows everybody and the landlords consider it safe to give credit to any laborer from this set of villages. Thus the $n_1$ and $n_2$ chosen in period 1 become parameters in the second period in the fragmented credit market. Moreover $n_1$ and $n_2$ have the same significance as the $n_1$ and $n_2$ in sections 2 and 3 above since landlord $i$ can lend to $n_1+n_2$ laborers where $n_3 = n - n_1 - n_2$.

It is immediately clear that in this model landlord $i$ may hire employees not just to work as laborers but keeping in mind that a larger $n_i$ alters the kind of leverage he has in the credit market. Hence this theory provides a rationale for interlinkage, albeit of a very different kind from the ones found in the literature [e.g., Braverman and Srinivasan (1981), Braverman and Stiglitz (1982), Basu (1983), Mitra (1983), Bell (1988)].

The natural solution criterion to use in such a two-period model is that of sub-game perfection. We shall first give an abstract characterization of this and then scrutinize a special case.

Let landlord $i$'s production function be

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[^8]: We shall assume that the parameters of the model are such that firm 1's chosen $n_1$ and firm 2's chosen $n_2$ never sum to greater than $n$. 
To keep the model simple, we assume that laborers have access to other employment opportunities at an exogenous wage, \( w \). In period 2 each landlord earns profits from production and interest from the credit market depending on what Nash equilibrium emerges from a fragmented duopoly characterized by \( n_1 \) and \( n_2 \).

In order to state this more formally note that if in a fragmented duopoly \( n \) and \( c \) are fixed, the fragmented duopoly is entirely defined by \( (n_1, n_2) \).

Let us now define a **Nash equilibrium correspondence**, \( N \), as follows. For every pair of non-negative integers \( n_1 \) and \( n_2 \) such that \( n_1 + n_2 \leq n \),

\[ N(n_1, n_2) = \{ (q_1, q_2) \mid (q_1, q_2) \text{ is a Nash equilibrium in a fragmented duopoly defined by } (n_1, n_2) \} \]

A specification of which Nash equilibrium will occur in period 2 for each game \( (n_1, n_2) \) is a **selection**, \( f \), from the Nash equilibrium correspondence \( N \). That is, \( f(n_1, n_2) \) is an element of \( N(n_1, n_2) \), for all \( (n_1, n_2) \).

Given that \( c \) represents the opportunity cost of giving credit and \( p \) is the price of credit, we could use \( \pi_i \) as defined in section 2 to be \( i \)'s profit function in period 2.

For every selection, \( f \), from the correspondence \( N \), we can define each player's profit in the two-period game (assuming zero discounting) as

\[ \Omega_i(n_1, n_2, f) = X_i(n_i) - wn_i + \pi_i(f(n_1, n_2)), \quad i = 1, 2, \quad (13) \]

where the absence of price discrimination in the credit market implies that all laborers will pay the same price and hence that both landlords will pay their laborers the exogenous wage \( w \).

The triple \( (n_1^*, n_2^*, f^*) \) is a (subgame-) perfect equilibrium if and only if

\[ \Omega_1(n_1^*, n_2^*, f^*) \geq \Omega_1(n_1, n_2^*, f^*) \text{ for all } n_1, \quad \text{and} \]

\[ \Omega_2(n_1^*, n_2^*, f^*) \geq \Omega_2(n_1^*, n_2, f^*) \text{ for all } n_2. \]

Distorting terminology slightly, we may refer to \( (n_1^*, n_2^*) \) as a 'perfect equilibrium' if there exists an \( f^* \) such that \( (n_1^*, n_2^*, f^*) \) is a perfect equilibrium.

In this setting, laborers are fully rational and make their choices after evaluating the consequences of joining one or other of the captive segments of the market, as opposed to dealing in the contested segment. However, we assume that \( n \) is large and laborers do not collude, so that each takes the pattern of segmentation \( (n_1, n_2) \) as exogenously given. As noted above, the
assumption of a constant marginal cost of funds, $c$, for both landlords implies that the monopoly price, $p^m$, is independent of the size of each captive market. Hence, if the laborer conjectures that the market will contain no contested segmented (i.e., $n_1 + n_2 = n$), he will face a price $p^m$ from both landlords; and since the wage is exogenously given, he will therefore be indifferent between them. If, on the other hand, there is a contested segment, then in the absence of price discrimination, the same price will rule everywhere; and again he will be indifferent as to which segment he joins. That is to say, an interlinked wage and credit contract with either landlord will yield a laborer the same utility as an unbundled deal. Thus, although laborers are not strategic in the sense that the actual choice of $n_i$ is effectively in the hands of the landlords alone, they are fully rational. In brief we model laborers in the same way as consumers are modeled in oligopoly theory.

We analyze the perfect equilibria of the two-period game in the special case where the demand schedule of an individual consumer is linear over the relevant range of outcomes:

$$p = a - bq.$$  \hfill (14)

We assume that $n$ is so large and the marginal product of labor, i.e., $X'(n_i)$, falls so fast that landlords 1 and 2 will never choose $n_1$ and $n_2$ for which there is a Nash equilibrium where one firm abandons the contested segment totally. Hence, we could focus on the unique ‘interior’ Nash equilibrium that occurs for each relevant $(n_1, n_2)$. Let $f(n_1, n_2)$ refer to such a Nash equilibrium.

Using (5), (6) and (14), we get

$$\pi_i(f(n_1, n_2)) = \frac{(a-c)^2}{b} \cdot \frac{(n-n_i)n^2}{(3n-n_1-n_2)^2}, \quad i = 1, 2.$$  \hfill (15)

Hence, using $\bar{\Omega}_i(n_1, n_2)$ to denote $\Omega_i(n_1, n_2, f)$, we have

$$\bar{\Omega}_i(n_1, n_2) = [X_i(n_i) - wn_i] + \left[ \frac{(a-c)^2}{b} \cdot \frac{(n-n_i)n^2}{(3n-n_1-n_2)^2} \right], \quad i = 1, 2.$$  \hfill (16)

Clearly, if $n_1^*$ and $n_2^*$ are such that $n_1^*$ maximizes $\bar{\Omega}_1(n_1, n_2^*)$ and $n_2^*$ maximizes $\bar{\Omega}_2(n_1^*, n_2)$, then $(n_1^*, n_2^*)$ is a perfect equilibrium.

The first interesting feature of the perfect equilibrium to note is that in equilibrium each landlord will be employing labor up to a point where the wage rate exceeds the marginal product of labor. Let $\hat{n}_i$ be such that

$$X'(\hat{n}_i) = w.$$
It is easy to see that for all \( n_1 \) and all \( n_1 < n_1^* \), \( \Omega_1(n_1, n_2) > \Omega(n_1, n_2) \). At \( n_1 = n_1^* \), a further increase in \( n_1 \) causes \( \Omega_1 \) to rise since, at this point, the first expression within brackets in (16) is stationary and the second expression is rising. Hence, landlords always employ in excess of what pure marginal productivity and wage considerations would lead them to do. This result is quite in keeping with Klemperer’s (1987b) finding of heightened competition in the ‘first’ period. It could also be thought of as providing a rationale for the idea that landlords have a penchant for maintaining an excessive number of dependent laborers [see, e.g., Bhaduri (1983)]. In addition, this model gives some new insight into the phenomenon of disguised unemployment and surplus labor, since it is possible for marginal product to be not only less than \( w \), but even zero (if \( X_i' \) vanishes for finite \( n_i \)).

From the first-order conditions of maximizing \( \Omega_1(n_1, n_2) \) with respect to \( n_1 \) and \( \Omega_2(n_1, n_2) \) with respect to \( n_2 \) and denoting the equilibrium values with a star, we have

\[
X_1'(n_1^*) + \frac{2(a - c)^2 n^2(n - n_2^*)}{b(3n - n_1^* - n_2^*)^3} = w
\]

and

\[
X_2'(n_2^*) + \frac{2(a - c)^2 n^2(n - n_1^*)}{b(3n - n_1^* - n_2^*)^3} = w.
\]

These, in turn, imply

\[
\frac{X_1'(n_1^*) - w}{n - n_1^*} = \frac{X_2'(n_2^*) - w}{n - n_2^*}.
\]  \hspace{1cm} (17)

Hence, if \( n_2^* > n_1^* \), then \( X_1'(n_1^*) > X_2'(n_2^*) \), since from the reasoning above we know that \( X_i'(n_i^*) - w < 0 \), \( i = 1, 2 \). It is important to appreciate that this is true though the production functions of the two landlords need not be the same. If we use the extent of divergence of \( X_i'(n_i^*) \) from \( w \) as an index of production inefficiency, then what we have established is that larger farms (in terms of numbers of workers employed) are the ones exhibiting greater production inefficiency. Also, larger farms have larger shares of the contested segment of the credit market,\(^9\) and hence are larger overall.

5. Equilibrium in agrarian markets with price discrimination

If landlords can practice price discrimination, the laborer who accepts an

\(^9\)From (5) and (6), we get \( \left( q_1/q_2 \right) = \left( (n - n_1^*)/(n - n_2^*) \right) \). Substituting into (15) yields the required result.
interlinked contract by going into a captive segment of the credit market knows that he will be charged the monopoly price, \( p^m \), which exceeds that in the contested segment, \( p^o \), should one exist.\(^{10}\) Thus, in order to make an interlinked contract attractive to laborers, landlords will have to offer a wage premium, \( \delta \) say, in compensation for the higher rate of interest. Landlords are therefore constrained by the utility equivalence condition

\[
v(p^m, w + \delta) = v(p^o, w),
\]

where \( v(\cdot) \) is the indirect utility of a laborer.

In this case, (13) becomes

\[
\Omega_i(n_1, n_2, f) = [X_i(n_i) - (w + \delta)n_i] + \pi_i(f(n_1, n_2))
\]

with the reminder that \( f \) now pertains to the Nash equilibrium as in section 3 (i.e. with price discrimination allowed).

\[
\pi_i(f(n_1, n_2)) = n_i p^m - q(p^m) + p \left( \frac{q_1 + q_2}{n_3} \right) \cdot q_i \cdot c(n_i q(p^m) - q_i),
\]

where \((q_1, q_2) = f(n_1, n_2)\).

With a linear demand function,

\[
p^m = (a + c)/2 \quad \text{and} \quad q^m = (a - c)/2b.
\]

In a standard duopoly, with \( n_3 = n - n_1 - n_2 \) given exogenously,

\[
p^o = (a + 2c)/3 \quad \text{and} \quad q^o = (q^1 + q^2)/n_3 = 2(a - c)/3b,
\]

and by symmetry,

\[
q^1 = q^2 = n_3 q^o/2.
\]

Substituting for \((p^m, q^m, p^o, q^1, q^2)\) in (19), some manipulation yields

\[
\pi_i(f(n_1, n_2)) = \frac{(a-c)^2}{b} \left( \frac{5n_i - 4n_j}{36} + \frac{n}{9} \right) \quad i = 1, 2, \quad i \neq j,
\]

which, unlike (15), is linear in \((n_1, n_2)\).

\(^{10}\)This argument uses subgame perfection which rules out the possibility of landlords committing themselves to some price different from \( p^m \).
The next step is to obtain the wage premium $\delta$ from (18). While the value of $\delta$ depends on $\pi(\cdot)$, it follows at once from the fact that $(p_m, p^*, w)$ are all independent of $(n_1, n_2)$ that $\delta$ must be likewise. Hence, substituting for $\pi_i(\cdot)$ from (20) in (13'), we have

$$\Omega_i(n_1, n_2) = [X_i(n_i) - (w + \delta)n_i] + \frac{(a - c)^2}{b} \left( \frac{5n_i - 4n_j}{36} + \frac{n}{9} \right), \quad i = 1, 2. \quad (21)$$

As in section 4, in equilibrium each landlord will be employing labor up to a point where the wage rate (including the premium $\delta$ in this case) exceeds the marginal product of labor. For when $[X_i(n_i) - (w + \delta)n_i]$ is stationary, $\pi_i$ is increasing in $n_i$. Denoting the values of $(n_1, n_2)$ in equilibrium with a star, we have, from (21),

$$X'_i(n^*_i) = (w + \delta) - \frac{5}{36} \frac{(a - c)^2}{b}, \quad i = 1, 2. \quad (22)$$

Hence, unlike the case without price discrimination, the marginal product of labor is identical on the farms of both landlords.

This result is not very surprising in the light of the fact that the assumption of constant marginal costs makes all the parameters of an individual's wage and credit contracts independent of the pattern of market segmentation, if landlords can practice price discrimination. As we saw in section 3, the cost functions (in this case, for lending) must be increasing and convex for interesting situations to arise from the one-period game with $(n_1, n_2)$ fixed. It is certainly plausible that the cost of lending, for example, is increasing and convex with the size of the captive market, since the landlord must prevent each of his captive clients from borrowing from the other landlord in period 2 and recover monopoly interest charges from them subsequently.

6. Rent dissipation

There remains the question of whether competition for captive segments of the market in period 1 will more than dissipate the rents from lock-in in period 2 [Klemperer (1987a)]. Suppose, therefore, that interlinking was banned. In a standard duopoly with $n_1 = n_2 = 0$ and $n_3 = n$, the profit of each landlord from moneylending is, under the above assumptions about costs and demand,

$$\pi^0_i = n(a - c)^2/9b. \quad (23)$$
In this case, the total profit of landlord \( i \) in equilibrium is

\[
\Omega^i_0 = [X_i(\hat{n}_i) - w\hat{n}_i] + n(a-c)^2/9b,
\]

where \( X_i'(\hat{n}_i) = w \). Subtracting (24) from (21), we obtain

\[
\hat{\Omega}_i(n_i^*, n_j^*) - \Omega^i_0 = [X_i(n_i^*) - wn_i^*] - [X_i(\hat{n}_i) - w\hat{n}_i] - \delta n_i^*
\]

\[+ (a-c)^2 \left( \frac{5n_i^* - 4n_j^*}{36} \right), \quad i = 1, 2. \quad (25)\]

Since \( \hat{n}_i \) maximizes \([X_i(n_i) - wn_i]\),

\[
\xi_i \equiv [X_i(n_i^*) - wn_i^*] - [X_i(\hat{n}_i) - w\hat{n}_i] < 0
\]

and the (algebraic) sum of the first three terms on the right-hand side of (25) is negative.

Now suppose that one landlord (1, say) has more land than the other; so that, by virtue of (22) and an assumption that land and labor are complementary, \( n_1^* > n_2^* \). Now, if the difference in holdings is such that \( n_2^* \leq 4n_1^*/5 \), it follows at once that the landlord who has the smaller holding would be better off if interlinking were banned.

In order to examine whether the combined rents from lock in of both landlords are more than fully dissipated by heightened competition for captive segments in period 1, we sum over \( i \) in (25) and obtain

\[
\sum_i \left[ \hat{\Omega}_i(n_i^*, n_j^*) - \Omega^i_0(n_1, n_2) \right] = (\xi_1 + \xi_2) - (n_1^* + n_2^*) \left[ \delta - (a-c)^2/36b \right]. \quad (26)
\]

Since \( \xi_i < 0 \), a strongly sufficient condition for the said rents to be more than dissipated is

\[
\delta > (a-c)^2/36b. \quad (27)
\]

Now the loss in an individual’s net consumer surplus that results from being charged the monopoly price as opposed to the duopoly price is

\[
(p^m - p^o)(q^m + q^o)/2 = 7(a-c)^2/72b > (a-c)^2/36b.
\]

Hence, as \( \delta \) is the compensating variation with respect to the increase in price from \( p^o \) to \( p^m \), (27) will indeed hold if consumption in each period is a non-inferior good for a laborer. We have therefore shown that rents from
second period lock-in can be more than dissipated in our model, a possibility which appears in other models in the related literature.

By way of comparison, we now examine whether this result will hold if landlords cannot practice price discrimination. Summing (18) over $i$ and subtracting $(\Omega_i^1 + \Omega_i^2)$, we obtain, in this case,

$$
\sum_i [\Omega_i(n_1^*, n_2^*) - \Omega_i^o(n_1, n_2)] = (\xi_1 + \xi_2) + \frac{(a-c)^2 n}{b} \left[ \frac{n(n+n_3^*)}{(2n+n_3^*)^2} - \frac{1}{9} \right].
$$

(28)

Since $9n(n+n_3^*) > (2n+n_3^*)^2$, strong claims about whether rents are more than fully dissipated cannot be made without knowledge of the shape of $X_i(n_i)$ over the domain $(\hat{n}_i, n_3^*)$, all of which determine the magnitude of $\xi_i$. We leave this as an open question.

7. Conclusion

This paper started by analyzing a market structure in which each firm has a predetermined set of potential customers. These sets may overlap but they need not coincide totally with one another. Such a structure could emerge in a location-model of oligopoly, but it emerges more naturally in trades where the problem of asymmetric information and moral hazard is high. Such markets were referred to as fragmented oligopolies, and the basic properties of a fragmented duopoly were analyzed formally. The next step was to make each firm's set of potential customers endogenous by embedding a fragmented duopoly in a two-period model and then examine its perfect equilibrium. This was done in the context of labor and credit markets in backward agrarian economies. There emerged a rationale for interlinking, albeit of a sort quite different from that advanced in the extant literature on that subject.

Fragmented oligopolies, it was argued, are relevant in a wide variety of situations. The particular model constructed in this paper was meant to be illustrative. By considering alternative strategy sets for firms and different solution concepts, a range of different models of fragmented oligopoly can be constructed. There is, in brief, room for much further exploration.

Appendix: Proof of Theorem 1

Given the symmetric nature of the two parts of Theorem 1, it is clearly sufficient to prove either.

From the second-order condition we have
\[ \frac{\partial^2 \pi_1}{\partial q_1^2} = \frac{2}{n_3} p' \left( 1 + \frac{n_1}{n_3} \right) + \frac{p''}{n_3^2} \left[ \left( 1 + \frac{n_1}{n_3} \right) q_1 + \frac{n_1}{n_3} q_2 \right] < 0. \] (A.1)

Taking total differentials in (5), we get

\[ p' \left( \frac{dq_1}{n_3} + \frac{dq_2}{n_3} \right) \left( 1 + \frac{n_1}{n_3} \right) + \frac{p''}{n_3^2} \left( \frac{dq_1}{n_3} + \frac{dq_2}{n_3} \right) \left[ \left( 1 + \frac{n_1}{n_3} \right) q_1 + \frac{n_1}{n_3} q_2 \right] \]

\[ + \frac{p'}{n_3} \left[ \left( 1 + \frac{n_1}{n_3} \right) dq_1 + \frac{n_1}{n_3} dq_2 \right] = 0. \]

Rearranging terms this may be rewritten as

\[ \frac{dq_1}{dq_2} = \frac{2}{n_3} p' \left( 1 + \frac{n_1}{n_3} \right) + \frac{p''}{n_3^2} \left[ \left( 1 + \frac{n_1}{n_3} \right) q_1 + \frac{n_1}{n_3} q_2 \right] - \frac{p'}{n_3}. \] (A.2)

Given (A.1) and \( p' < 0 \), it follows that \( (dq_1/dq_2) > -1 \). Since (5) implicitly defines the reaction function \( R_1(\cdot) \), we have proved Theorem 1.

References